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LETTER TO THE EDITOR

Asynchronous excitation of a plasma instability

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Abstract. Results are presented which show that the classical effect of 'asynchronous excitation of an oscillator' can be applied to the excitation of an ion sound instability in a plasma. Further, the predicted upper and lower threshold values of the asynchronous driving amplitude are observed, for the region in which the instability is excited.

Previous work (Keen and Fletcher 1970a, 1972) has shown that the behaviour of an ion sound instability in a plasma can be described well by a nonlinear equation of the Van der Pol type (Van der Pol 1922). In particular, a previous experiment has shown that an existing ion sound instability in a plasma could be suppressed by the method 'asynchronous quenching of a Van der Pol oscillator' (Keen and Fletcher 1970b). In that method, the density perturbations of the ion sound instability at frequency ω_0 were driven by an external oscillator at frequency $\omega(\gg \omega_0)$ and as the driving amplitude was increased the system behaved as if the asynchronous action of this frequency (ω) were destroying or 'quenching' the previously existing selfexcited oscillation. In this paper, the converse effect is described in which the ion sound instability does not exist, until the externally induced high frequency (ω) oscillations are applied to the plasma system. The effect of the external signal is to virtually modify the damping of the instability, until the effective damping becomes negative and the instability can grow. This is termed asynchronous excitation of an oscillator or instability (Minorsky 1947).

In Keen and Fletcher (1970a, 1972) the plasma stability was investigated using the two fluid model, to describe the motion of the ions and electrons separately. Included in the ion equation of continuity, was an ion source term $S(n, T_e, \bar{E})$, caused by the presence of large amplitude density fluctuations in the plasma, which created plasma locally by local heating effects, local ionization, etc. Consequently, this source term is a function of local density n, electron temperature T_e, and local electric field $\bar{E}$. It has been shown from thermodynamic arguments (Keen and Fletcher 1970a, 1972) that this source term is given by an expansion of the form

$$S_i = \alpha n_1 - \gamma n_1^3 + \epsilon n_1^5 - \dots \qquad (1)$$

where only odd terms have been kept since these are the only ones found to be effective. Using this source term, an equation of the Van der Pol type can be obtained, describing the temporal variation of the density perturbations n_1, given by

$$\frac{d^2n_1}{dt^2}+\frac{dn_1}{dt}\{(\nu-\alpha)+3\gamma n_1{}^2-5\epsilon n_1{}^4\}+\omega_0{}^2n_1=0 \tag{2}$$

where $\omega_0 = k_z c_s \gg \alpha$, $\epsilon n_1{}^4$ and $\gamma n_1{}^2$, $c_s = (T_e/M)^{1/2}$, ω_0 is the ion sound instability frequency, and ν is the ion–neutral collision frequency. Equation (2) can be put in the form

$$\frac{d^2n_1}{dt^2}+\omega_0{}^2n_1=f\left(n_1,\frac{dn_1}{dt}\right). \tag{3}$$

If we consider the applications of high frequency oscillations ($\omega \gg \omega_0$) on this system, equation (3) can be solved by the method of equivalent linearization (Minorsky 1947). Here equation (3) is put in the form

$$\ddot{n}_1+(\omega_0{}^2+\bar{K})n_1=\bar{\lambda}\dot{n}_1 \tag{4}$$

where

$$\bar{\lambda}=\frac{1}{2\pi^2a_0\omega_0}\int_0^{2\pi}\int_0^{2\pi}f\{(a+b),(\dot{a}+\dot{b})\}\sin\theta\,d\theta\,d\phi \tag{5}$$

and

$$\bar{K}=-\frac{1}{2\pi^2a_0}\int_0^{2\pi}\int_0^{2\pi}f\{(a+b),(\dot{a}+\dot{b})\}\cos\theta\,d\theta\,d\phi \tag{6}$$

where a trial solution of the form $n_1 = a+b = a_0 \cos\theta + b_0 \cos\phi$ has been assumed, with $\theta = \omega_0 t$ and $\phi = \omega t$.

It can be seen from equation (4), that if $\bar{\lambda} > 0$ we have growth of the instability, and for $\bar{\lambda} < 0$ the ion wave is damped out.

By substituting for $f(n_1, \dot{n}_1)$ from equation (2), $\bar{\lambda}$ can be calculated to give

$$\bar{\lambda}(a_0,b_0) = -(\nu-\alpha)+\tfrac{3}{4}\gamma a_0{}^2+\tfrac{3}{2}\gamma b_0{}^2-\tfrac{5}{8}\epsilon a_0{}^4-\tfrac{15}{4}\epsilon a_0{}^2b_0{}^2-\tfrac{15}{8}\epsilon b_0{}^4. \tag{7}$$

When the high frequency (ω) (heteroperiodic) excitation is absent, $b_0 = 0$, and equation (7) becomes

$$\bar{\lambda}(a_0,0) = -(\nu-\alpha)+\tfrac{3}{4}\gamma a_0{}^2-\tfrac{5}{8}\epsilon a_0{}^4 \tag{8}$$

and this has a maximum value, where $a_0{}^2 = 3\gamma/5\epsilon$. This gives a maximum value for $\bar{\lambda}_{\max}(a_0, 0) = -(\nu-\alpha)+9\gamma^2/40\epsilon$, which means that when $\bar{\lambda}_{\max} \simeq 0$ the instability is just damped out (marginal instability). Then for asynchronous excitation of the instability, the following condition must be satisfied:

$$\bar{\lambda}(0,b_0)-\bar{\lambda}_{\max}=\tfrac{3}{2}\gamma b_0{}^2+\tfrac{15}{8}\epsilon b_0{}^4-\tfrac{9}{40}\gamma^2/\epsilon>0 \tag{9}$$

which is satisfied when

$$\frac{\gamma}{5\epsilon}<b_0{}^2<\frac{3\gamma}{5\epsilon}. \tag{10}$$

Consequently, there is a lower threshold ${}_{l}b_0$ and an upper threshold value ${}_{u}b_0$, for asynchronous excitation of the instability to occur, the ratio of which is ${}_{\mathrm{u}}b_0{}^2 : {}_{\mathrm{l}}b_0{}^2 = 3 : 1$.

In the experiments, the plasma used was the positive column of a neon arc discharge ($n \simeq 2\times10^{11}\,\mathrm{cm}^{-3}$ and $T_e = 5{\cdot}4\,\mathrm{eV}$), and has been described previously (Keen and Fletcher 1970a, 1972). Initially, in this plasma, a selfexcited oscillation was present, which has been identified as an ion sound instability (Keen and Fletcher 1970a, 1972). It was observed with predominantly a single frequency, $\omega_0 \simeq 7{\cdot}1$ kHz, and was found to have an $m = 0$ azimuthal mode and to be a standing wave in the axial direction with a wavelength $\lambda_z = 80$ cm. After experiments to identify its nature, the instability was reduced to its marginal state by increasing the neutral pressure, thus increasing the ion–neutral collision frequency ν, until the oscillation was just damped out. It was in this marginal state that the experiment was performed.

External signals in the range 20–100 kHz were coupled to the plasma from a small magnetic coil wound around the glass discharge tube. An AC current at the desired frequency ω through the coil produced density oscillations at this frequency in the plasma. The effect on the plasma of this driving oscillation at ω was monitored by observing the output from an ion-biased probe, on a spectrum analyser. This allowed the instability amplitude a_0 as well as the driven amplitude b_0 at ω to be measured simultaneously.

The results obtained at two drive frequencies, $\omega = 30$ kHz and $\omega = 50$ kHz are shown in figure 1, where the square of instability amplitude $a_0{}^2$ at ω_0 is plotted against

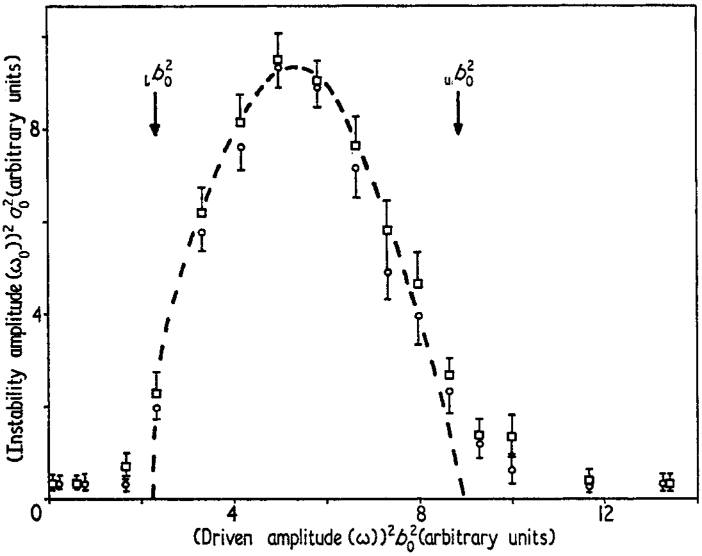

Figure 1. Plot of the square of the instability amplitude $a_0{}^2$ against the square of the driven amplitude $b_0{}^2$ at two frequencies: ○ $\omega = 30$ kHz; □ $\omega = 50$ kHz.

the square of the driven amplitude $b_0{}^2$, at ω_0. It is seen that at small driven values there is no instability in the plasma but at a certain threshold value ${}_{\mathrm{l}}b_0$ the instability appears, and increases in amplitude as b_0 is increased, until a maximum value of a_0 is reached. Further increasing b_0, only reduces a_0, until an upper threshold value of $b_0 = {}_{\mathrm{u}}b_0$ is achieved, and at this point the instability disappears again. Higher drive values above ${}_{\mathrm{u}}b_0$ produce no effect on the plasma, throughout the range of ω tried (20–100 kHz). The ratio of $({}_{\mathrm{u}}b_0/{}_{1}b_0)^2$ is $3{\cdot}8 \pm 0{\cdot}6$ in this experiment, which is in tolerable agreement with the expected value of 3.

Therefore, it is concluded that the instability effects observed in this experiment are due to asynchronous excitation of the ion sound instability.

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